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A new robust adaptive control of a class of MIMO nonlinear systems with fuzzy approximators

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Based on the Lyapunov synthesis approach, several adaptive fuzzy control schemes have been developed during the last few years. In this paper we develop a robust adaptive fuzzy control law for MIMO nonlinear system class. The proposed method uses the Sugeno-Takagi fuzzy system as an universal approximator of continuous nonlinear functions. The adaptive control law is established based on the Lyapunov method. So, the output convergence, the boundedness of the parameters and the states are derived. Moreover, the fuzzy adaptive law incorporates a compensatory sliding term, which compensates for effects of the unavoidable reconstruction errors.

Key words: fuzzy systems, fuzzy adaptive law, sliding term, reconstruction Errors, permanent magnet synchronous motors, current controlled inverter

1. Introduction

In industrial field, the control synthesis is often carried out based on the theory of linear time invariant systems using the conventional PI or PID type regulators. These regulators are really efficient only around an operative point. Sometimes, the system under control is nonlinear time variant and its dynamics can be badly known or partially known. For these systems, the linear control exhibits generally poor performances and the recourse to a nonlinear adaptive control can be a judicious solution.

Besides, the theory of fuzzy logic has also been applied successfully for the control of nonlinear systems. In general the control strategy used for fuzzy logic controller is based on expert knowledge, so the fuzzy logic controller has the ability to emulate the human strategies control. Moreover, it would be necessary that the control strategy can perform the control objectives even if the parameters of the system evolve or are badly known. In order to solve this problem, we develop an adaptive fuzzy controller which is able to modify the control law according to the evolution of the system parameters.

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During the past two decades, many contributions have been devoted to the development of the fuzzy logic controllers [4-7] based on the Takagi-Sugeno (T-S) model or the fuzzy dynamic models. The basic idea of these methods is:

a. To represent the complex nonlinear system by a family of local linear models, each linear model exhibits the dynamics of the complex system in one local region. Then, to construct a global nonlinear model by aggregating the local models through the fuzzy membership functions.

b. To design the local controllers based mainly on each local model, which is much easier than on the global region for the nonlinear system. Then, the global controller can be aggregated from the local controllers.

It was also proven that the fuzzy system is capable of approximating any nonlinear functions over a convex compact region [8]. Based mainly on this property, the fuzzy logic system is applied in the area of the adaptive control where the unknown nonlinear functions are approximated by a fuzzy basis functions and its parameters are updated on line to cope with uncertainties.

Exploiting the universal approximation theorem, globally stable direct and indirect adaptive controllers are first synthesized using Lyapunov function [9]. Afterwards, several stable adaptive fuzzy control schemes are proposed for nonlinear systems [10-13]. Unfortunately, these schemes are restricted only to the SISO nonlinear systems. Recently, several contributions are devoted to the development of adaptive fuzzy controllers dedicated to the MIMO nonlinear systems. For this purpose, the structure of these controllers is generally based either on the indirect approach [14-17] or on the direct approach [16-19]. The class of MIMO systems considered, in these papers, is in the form:

$$y^{(r)} = F(x) + G(x)u \tag{1}$$

with $x \in \Re^n$ is the state vector, $u \in \Re^p$ is the control input, $y \in \Re^p$ is the output vector, $F(x) \in \Re^p$ and $G(x) \in \Re^{p \times p}$ are unknown nonlinear vector function and matrix function of control respectively. It should be noted that the developed work, concerning the adaptive fuzzy control of this class of nonlinear system, depends closely on the properties conceded with the $G(x)$ matrix function of control.

Indeed, the control law, exposed in [14], requires the knowledge of the $G(x)$ norm. Moreover this norm must be lower bounded, which is rather difficult to be satisfied in practice. On the other hand in [15], a projection algorithm is applied. Meanwhile this algorithm complicates the control law and the bounds of the parameters must be known. The determination of such bounds is not an easy task. So, to avoid this difficulty, it has been shown that the projection algorithm can be also applied whenever the matrix is strictly diagonal dominant or upper (lower) triangular [16]. Another alternative [17] is explored for the case where $G(x)$ is positive definite. But, the suggested control law is discontinuous and the upper bound of the norms of $F(x)$ and $G(x)$ must be given.

Concerning the direct approach, the control law is calculated without resorting to an approximation of the model related to the controlled system [16-19]. For the exposed direct approaches in [17, 18], the control matrix is assumed to be known and, therefore these approaches are not really connected with direct structure. But, for the real direct approach suggested in [16], the control matrix is assumed strictly diagonal dominant moreover the bounds of the derivative of its diagonal entries must be necessary known. While, the method proposed in [19] is applicable only when $G(x)$ is symmetrical and positive definite. Moreover, the control law imposes that the norm of the temporal derivative must be bounded by a continuous known function.

In this paper, we develop a new stable fuzzy adaptive control law for a class of MIMO nonlinear systems in order to confer high robustness to the controller in the presence of parametric uncertainties and dominant uncertain nonlinearities. It is important to note that the proposed control law 'relaxes' the necessity: first, to have $G(x)$ with a dominant diagonal or upper (lower) triangular and second, to know the bounds of the temporal derivative of its diagonal entries. Moreover, the update law of the parameters avoids the cumbersome projection algorithm. The fuzzy systems are used to approximate the model of controlled system. In order to compensate for the effects of the unavoidable reconstruction errors, we introduce a sliding term into the control law. The approximation theory and the Lyapunov method are used together to construct, in the first stage, the fuzzy adaptive control law and to establish, in the second stage, the convergence of the tracking error and the boundedness of the adaptive parameters and all plant signals. The method is applied in simulation to the problem of speed or position tracking of the permanent magnets synchronous motor (PMSM).

This paper is organized as follows: in section 2 the used fuzzy logic system is briefly described and section 3 is devoted to the problem statement. The proposed fuzzy adaptive scheme for a class of MIMO nonlinear system is developed in section 4. In section 5, the performances of the proposed scheme are evaluated by simulation for the case of PMSM. The conclusion is presented in section 6.

2. Description of the used fuzzy logic system

The fuzzy logic system incorporates generally four principal components: fuzzifier, fuzzy rules base, inference engine and defuzzifier [20-22]:

- Fuzzifier maps crisp points in the input space into fuzzy sets in the input space.
- Fuzzy rules base contains the fuzzy rules interpreting the behaviour of a given system; it is the central element since the other components interpret and combine these rules to form the final output.
- Inference engine exploits an approximate reasoning procedure in order to map fuzzy sets in the input space into fuzzy sets in the output space.
- Defuzzifier extracts crisp points in the output space from fuzzy sets in the output space. A FLS can be seen as a mathematical application of the input towards

the output. This application is very rich in its mathematical formulation by the existence of various mathematic interpretations concerning the fuzzy rules, the fuzzy inference and the defuzzifier.

It is significant to note that the implementation of the adaptive fuzzy control, in real time, requires that the mathematical model of the FLS must be simple. In our case, we are interested by the FLS of Sugeno-Takagi model, developed initially to model a system from numerical data [23]. In this case the consequences rules are numerical functions, which depend on the values of the crisp input variables. In this section the mathematical formulation of used fuzzy systems and fuzzy basis functions in the case of Sugeno-Takagi model is given. One notes by $x_{sf_1},\cdots x_{sf_n}$ the inputs of the FLS, and by y_{sf} its output. Each variable x_{sf_i} is related to m_i fuzzy sets F_i^j defined on U_i. Moreover, it is assumed that for any value of x_{sf_i} on U_i, there exist at least one fuzzy set among F_i^j ($i=1,2,\ldots,n$ and $j=1,2,\ldots,m_i$) for which the membership degree is not null. The rule base of the FLS incorporates $M=\prod_{i=1}^{n} m_i$ rules of the form:

$$\begin{aligned} R_l \ :\ & \text{IF } x_{sf_1} \text{ IS } F_1^{l_1} \text{ AND } \ldots \\ & \ldots\text{AND IF } x_{sf_n} \text{ IS } F_n^{l_n} \text{ THEN } y_{sf_l}(x) = a_0^l + a_1^l x_{sf_1} + \cdots + a_n^l x_{sf_n} \end{aligned} \quad (2)$$

with $l=1,\ldots,M$, $i=1,\ldots,n$, and $1\leqslant l_i\leqslant m_i$. Indeed, the base of fuzzy rules contains all the combinations of the fuzzy sets related to the input variables.

In this paper, we employ the product operation for fuzzy relation and T-norm. The definition of the product operation is the same as in [28]. Besides, the singleton fuzzifier and weighted average deffuzzification is used. The overall output value is:

$$y_{sf} = \frac{\sum_{l=1}^{M}\mu_l y_{sf_i}}{\sum_{l=1}^{M}\mu_l} \quad (3)$$

where μ_l stands for the firing strength of the R_l rule, which is given by:

$$\mu_l = \prod_{i=1}^{n}\mu_{F_i^{l_i}}(x_i); \quad 1\leqslant l_i\leqslant m_i \quad (4)$$

and $\mu_{F_i^{l_j}}(x_i)$ is the membership function of variable x_i associated to fuzzy set $\mu_{F_i^{l_i}}$. This function is selected as Gaussian function:

$$\mu_{F_i^{l_j}}(x_{sf_i}) = exp\{-0.5(v_i^j(x_{sf_i}-c_i^j))\} \quad (5)$$

where c is the average, v is the inverse of the variance.

If the premises parameters are fixed a'priori thus only the conclusion parameters can be freely adjustable. Thus, the final output can be rewritten under the form:

$$y_{sf} = W(x_{sf})A \quad (6)$$

where A is a vector gathering the parameters a_i^j and $W(x_{sf})$ is a vector of basis fuzzy functions.

3. Problem statement

The problem arising, relates to the development of a direct adaptive control based on the FLS for a class of MIMO nonlinear system, this class is in the form:

$$u = F(X)x^{(n)} + G(X)$$
$$y = x \tag{7}$$

where $x = [x_1, \ldots, x_n]^T$ and the notation $x^{(n)}$ stands for the nth order time derivative of the variable x. The vectors $X = [(x^{(n-1)})^T, \ldots, x^T]$, $u = [u_1, \ldots, u_m]^T$ and $y = [y_1, \ldots, y_m]^T$ are the states, the control input and the plant output respectively. $F(X) \in \Re^{m\times m}$ is a vector of nonlinear functions whereas $G(X) \in \Re^m$ is a matrix of nonlinear functions. The control objective is to force the output vector $x = [x_1, \ldots, x_m]^T$ to follow the specified desired trajectory $x_d = [x_{d1}, \ldots, x_{dm}]$. Define the tracking error vector $e(t)$ as:

$$e(t) = x(t) - x_d(t). \tag{8}$$

Therefore, we should design a fuzzy adaptive control law $u(t)$ such that $e(t)$ converges to a small neighborhood of zero. To this end, the following assumptions are assumed:

Assumption A1.

- $F(x) \in \Re^{m\times m}$ and $G(x) \in \Re^m$ are bounded smooth nonlinear functions.
- The state vector X is available.
- The reference signal x_d and its derivatives $(\dot{x}_d, \ddot{x}_d, \ldots, x_d^{(m)})$ are known bounded signals.

If the functions $F(X)$ and $G(X)$ are well known the control input u^* can be taken as [26]:

$$u^* = F(X)v + G(X) \tag{9}$$

with

$$v = x_d^{(n)} + k_n e^{(n-1)} + \cdots + k_1 e, \tag{10}$$

$$e = x_d - x, \quad k_i > 0, \quad i = 1, \ldots, n. \tag{11}$$

Introducing (9) into (7), leads to the tracking error dynamic equation:

$$e^{(n)} + k_n e^{(n-1)} + \cdots + k_1 e = 0. \tag{12}$$

Since the coefficients $k_1, \ldots, k_n$ are imposed such that $p^{(n)} + k_n p^{(n-1)} + \cdots + k_1$ polynomial is Hurwitz, therefore the tracking error vector $e(t)$ converges asymptotically to zero. In the case where the functions $F(X)$ and $G(X)$, involved in the dynamic model (7), are badly known, the implementation of the control law (9) is inoperative since it requires a precise model. To solve this problem an approach by an FLS is proposed. For this purpose, the dynamic of the system is rewritten, first of all, in the following form:

$$F(X)x^{(n)} + G(X) = f(X, x^{(n)}). \tag{13}$$

Let $\hat{f}(X, x^{(n)}, \hat{\theta}_f)$ be the estimated function $f(X, x^{(n)})$ where $\hat{\theta}_f$ is a parameter vector. It is possible to extract the functions $\hat{F}$ and $\hat{G}$ from $\hat{f}(X, x^{(n)}, \hat{\theta}_f)$ such that:

$$\hat{f}(X, x^{(n)}, \hat{\theta}_f) = \hat{F}(X)x^{(n)} + \hat{G}(X). \tag{14}$$

Therefore, one can construct the following control input $u(t)$:

$$u(t) = \hat{F}(X)(v + u_{gl}) + \hat{G}(X) \tag{15}$$

where u_{gl} is a sliding term which will be clarified later. The control law (15) requires an FLS for reconstructing the function $\hat{f}(X, x^{(n)}, \hat{\theta}_f)$ and an adaptation mechanism for the parameters $\hat{\theta}_f$ so that this control law ensures the convergence of the tracking error $e(t)$ to zero and the boundedness of all signals of the plant.

4. Control synthesis and stability analysis

The FLS is principally used to estimate on-line the nonlinear function given in (13). To this end, the function $f(\cdot)$ is ideally approximated by an FLS such that:

$$f(X, x^{(n)}) = W_f(X, x^{(n)})\theta_f + \varepsilon_f \tag{16}$$

where $W_f(X, x^{(n)})$ is a basis function [20] and θ_f is vector of optimal parameters, while ε_f is the unavoidable reconstruction error satisfying the condition [27, 28]:

$$||\varepsilon_f|| \leqslant \bar{\varepsilon}_f, \quad \bar{\varepsilon}_f > 0, \quad 1 \leqslant i \leqslant m. \tag{17}$$

In consequence, the function $\hat{f}(X, x^{(n)})$ which is the approximation of $f(X, x^{(n)})$ can be defined under the form:

$$\hat{f}(X, x^{(n)}) = W_f(X, x^{(n)})\hat{\theta}_f \tag{18}$$

where $\hat{\theta}_f$ is the estimate of the best parameters θ_f. The estimated function $\hat{f}(\cdot)$ is decomposed as follows:

$$\hat{f}(X, x^{(n)}) = \hat{F}(X)x^{(n)} + \hat{G}(X). \tag{19}$$

Define the filtered error vector $s = [s_1, \ldots, s_m]$ as:

$$s_i(t) = \left(\frac{\partial}{\partial t} + \lambda_i\right)^{n-1} e_i; \quad \lambda_i \geqslant 0; \quad i = 1, \ldots, m. \tag{20}$$

The coefficient λ_i is selected so that the transfer function $H_i(p)$, $i = 1, \ldots, m$:

$$H_i(p) = \frac{p + \lambda_i}{p^{(n)} + k_n p^{(n-1)} + \cdots + k_1} \tag{21}$$

that is to say strictly real positive.

Proposition *While having the fuzzy model (14), under the assumptions A1, and if the system (7) is conducted by the control law*

$$u = \hat{F}(X)(v + u_{gl}) + \hat{G}(X) \tag{22}$$

where

$$u_{gl} = k_{gl}\,\mathrm{sign}(s)||\hat{F}^{-1}|| \tag{23}$$

$$k_{gl} = \bar{\varepsilon}_f \tag{24}$$

with

$$\mathrm{sign}(s) = \mathrm{diag}(\mathrm{sign}(s_i)); \quad i = 1, \ldots, m \tag{25}$$

and the parameters $\hat{\theta}_j$ are updated under the law:

$$\dot{\hat{\theta}} = \Gamma W_f^T(X, x^{(n)})(\hat{F}^{-1})^T s \tag{26}$$

where Γ is a positive definite matrix. Therefore, the tracking error converges asymptotically to zero and the state vector X and the parameters $\hat{\theta}_j$ are bounded.

The proposed adaptive control, which adaptive mechanism is based on the fuzzy logic system, is illustrated by the control structure related to the Fig. 1.

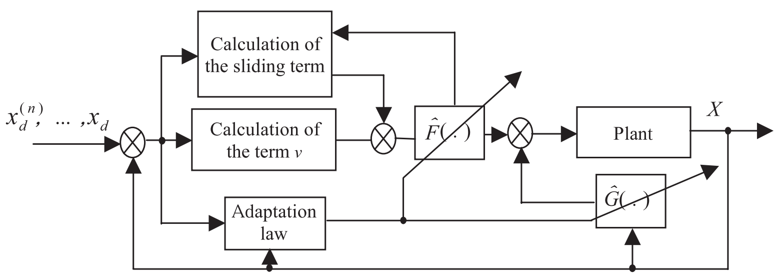

Figure 1. Structure of the direct fuzzy adaptive control

PROOF. By introducing the control law (22) into the equation (7), the dynamics of the tracking error becomes:

$$e^{(n)} = -k_n e^{(n-1)} - \cdots - k_1 e + E_{ex} \tag{27}$$

with $E_{ex} = [E_{ex}(1), \dots, E_{ex}(m)]$ is given by:

$$E_{ex} = -\hat{F}(X)^{-1}\left\{\hat{F}(X)x^{(n)} + \hat{G}(X) - F(X)x^{(n)} - G(X)\right\} - u_{gl}. \tag{28}$$

Therefore the error dynamic e_i and the filtered error s_i are:

$$\begin{cases} e_i^{(n)} = -k_n e_i^{(n-1)} - \cdots - k_1 e_i + E_{ex}(i) \\ s_i = \left(\frac{\partial}{\partial t} + \lambda_i\right)^{n-1} e_i; \quad i = 1, \dots, m. \end{cases} \tag{29}$$

We introduce the vector $y_i = [e_i, \dots, e_i^{(n-1)}]^T$ which leads to:

$$\begin{cases} \dot{y}_i = A_i y_i + B_i E_{ex}(i) \\ s_i = C_i^T y_i \quad i = 1, \dots, m \end{cases} \tag{30}$$

with, for $i = 1, \dots, m$ one has:

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -k_1 & -k_2 & -k_3 & -k_4 & \cdots & -k_n \end{bmatrix},$$
$$B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C_i = [\lambda^{n-1}, \ (n-1)\lambda^{n-2}, \dots, (n-1)\lambda, 1]. \tag{31}$$

Since, the transfer functions $H_i(p)$, $i = 1, \dots, m$ are strictly real positive, consequently there exist symmetrical positive definite matrices P_i and Q_i such that:

$$\begin{cases} A_i^T P_i + P_i A_i = -Q_i \\ P_i B_i = C_i^T \quad i = 1, \dots, m. \end{cases} \tag{32}$$

The equations (30) for $i = 1, \dots, m$ can be grouped in the following compact form:

$$\begin{cases} \dot{y} = Ay + BE_{ex} \\ s = C^T y, \quad i = 1, \dots, m \end{cases} \tag{33a}$$

with

$$\begin{cases} y = [y_1 \ \ y_2 \ \ \dots \ \ y_m]^T, \quad s = [s_1 \ \ s_2 \ \ \dots \ \ s_m]^T \\ A = \operatorname{diag}[A_i], \ B = \operatorname{diag}[B_i], \ C^T = \operatorname{diag}[C_i^T], \quad i = 1, \dots, m. \end{cases} \tag{33b}$$

The equalities matrices associated with the system (33) and deduced from (32) are then:

$$\begin{cases} A^T P + PA = -Q \\ PB = C^T \quad i = 1, \dots, m \end{cases} \tag{34}$$

with

$$P = \operatorname{diag}[P_i], \quad Q = \operatorname{diag}[Q_i], \quad i = 1, \dots, m. \tag{35}$$

The stability analysis of the closed loop system is carried out by using the candidate Lyapunov function:

$$V = Y^T PY + \widetilde{\theta}_f^T \Gamma^{-1} \widetilde{\theta}_f \tag{36}$$

where $\widetilde{\theta}_f$ stands for the parametric error which is defined by:

$$\widetilde{\theta}_f = \hat{\theta}_f - \theta_f. \tag{37}$$

The time derivative of the Lyapunov function V is then:

$$\dot{V} = \dot{Y}^T PY + Y^T P\dot{Y} + 2\widetilde{\theta}_f^T \Gamma^{-1} \dot{\hat{\theta}}_f. \tag{38}$$

If, in the relation (38), $\dot{Y}$ is substituted by its dynamic (33a), and by taking account to the matrices equalities (34), this one becomes:

$$\dot{V} = -Y^T QY + 2s^T \hat{F}^{-1} \overline{\varepsilon}_f - 2s^T u_{gl} - 2s^T \hat{F}^{-1} W_f(X, x^{(n)}) \widetilde{\theta}_f + 2\widetilde{\theta}_f^T \Gamma^{-1} \dot{\hat{\theta}}_f. \tag{39}$$

Introducing the parameters adaptive law (26) into (39), $\dot{V}$ is reduced to:

$$\dot{V} = -Y^T QY + 2s^T \hat{F}^{-1} \overline{\varepsilon}_f - 2s^T u_{gl}. \tag{40}$$

Moreover, the following inequality is always fulfilled:

$$\dot{V} \leqslant -Y^T QY + 2|s^T||\hat{F}^{-1}| \overline{\varepsilon}_f - 2s^T u_{gl}. \tag{41}$$

At last, one replaces the term of the sliding mode u_{gl} by its expression (23), which leads to the following inequality:

$$\dot{V} \leqslant -Y^T QY. \tag{42}$$

Since, the matrix Q is positive definite the condition (42) guarantees of the Lypunov stability. Hence, the control law (22) and the parameters adaptation law (26) ensure together the asymptotic convergence of the tracking error vector Y towards zero and the boundedness of the parameters $\hat{\theta}$ and the state vector x.

Remark 1: The adaptation law (26) updates only the conclusion parameters. Indeed in our case, the designer specifies, in advance, the structure of FLS, the input variables, the fuzzy sets (or membership functions) and the number of rules. In practice, to make the "good choice" for all these FLS parameters, in advance, is a difficult task, apart for a skilled operator in the area of the controlled system. A common practice is to arbitrary define the membership functions to cover the interest subset of the input space [9, 30, 31, 32]. One can think that this adaptation law also compensates, in a certain manner, for the inadequacy of the fuzzy sets and the insufficiency of the rules number.

5. Application to permanent magnet synchronous motor

5.1. Mathematical model of PMSM

The model of the permanent magnet synchronous motors (PMSM) is considered in the case of the usually allowed simplifying assumptions i.e.:

- The spatial distribution of stator winding is sinusoidal.
- The saturation is neglected.
- The damping effect is neglected.

Thus, in the synchronous d-q reference form, the dynamic of PMSM is represented as follows:

$$v_d = R_s i_d + L_d \frac{di_d}{dt} - pL_q \Omega i_q, \tag{43}$$

$$v_q = R_s i_q + L_q \frac{di_q}{dt} + pL_d \Omega i_d + p\Omega\Phi_f, \tag{44}$$

$$J\frac{\Omega}{dt} = T_{em} - T_r - F_c\Omega, \tag{45}$$

$$T_{em} = \frac{3}{2} p(\Phi_f i_q + (L_d - L_q) i_d i_q) \tag{46}$$

where v_d, v_q – stator voltage in d-q-axis, i_d, i_q – stator current in d-q-axis, L_d, L_q – stator inductance in d-q-axis, R_s – stator resistance, p – number of pole pairs, Ω – mechanical speed of motor, Φ_f – flux created by the rotor magnets, T_{em}, T_r – electromagnetic torque and load torque, F_c – viscous friction coefficient, J – total moment of inertia of the motor and load.

5.2. Speed control

In the case of surface-mounted PMSM ($L_d = L_q$), the electromagnetic torque depends solely on the current in the q axis. For a given torque, the transferred power is optimized if the current in the direct axis is null ($i_d = 0$) [29]. Hence, the control objective is to force the current i_d to zero and to impose the demanded torque by controlling the current i_q. Physically by this strategy, the linked stator flux is maintained in quadrature with flux produced by the rotor magnets. The proposed scheme of direct adaptive fuzzy control, about the speed tracking of the PMSM, appears in Fig. 2.

From the reference speed Ω_{ref} and measured speed Ω, the fuzzy adaptive controller provides the desired current i_{qref}. The three-phase reference current is obtained from the (d-q) stator reference current $(i_{qref}, i_{dref} = 0)$ by using the inverter Park transformation. The actual stator current (i_a, i_b, i_c) is restricted in hysteresis bandwidth Δi around the three-phase reference currents by using an appropriate switching of the inverter legs. By

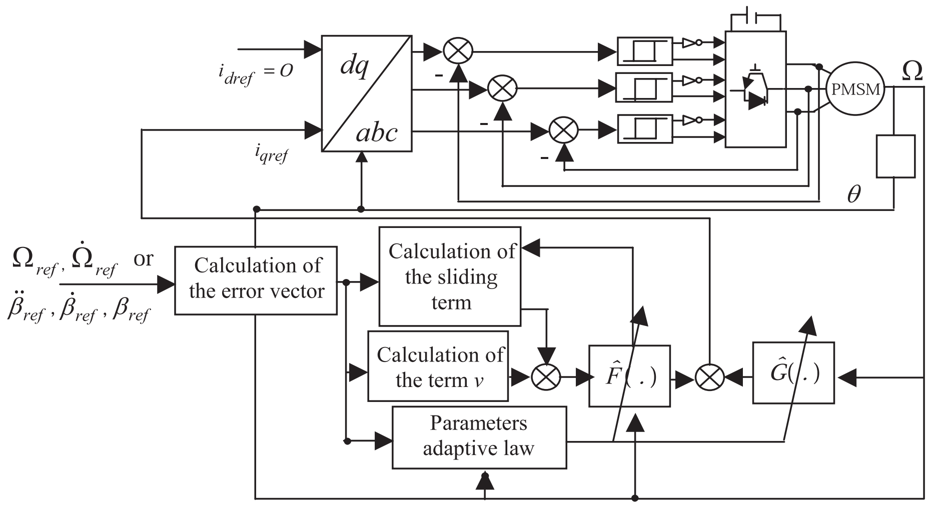

Figure 2. Direct adaptive fuzzy control scheme of permanent magnet motor

using the equilibrium equation relating the motor torque and the torque opposed by the mechanical part of the system, we can write:

$$i_q = F(\Omega)\frac{d\Omega}{dt} + G(\Omega). \tag{47}$$

In the purpose of facilitating the synthesis of the adaptive fuzzy system, it is more convenient to write the 2nd member of the equation (47) under the following compact form:

$$F(\Omega)\frac{d\Omega}{dt} + G(\Omega) = f(\Omega, \dot{\Omega}). \tag{48}$$

The function $f(\Omega, \dot{\Omega})$ can be rebuilt perfectly by using the fuzzy logic system:

$$f(\Omega, \dot{\Omega}) = W_f(\Omega, \dot{\Omega})\theta_f + \varepsilon_f \tag{49}$$

where θ_f are the best (or optimal) parameters. However, the estimated $\hat{f}(\cdot)$ of the function $f(\cdot)$ can by generated only in the form:

$$\hat{f}(\Omega, \dot{\Omega}) = W_f(\Omega, \dot{\Omega})\hat{\theta}_f. \tag{50}$$

For this application, the fuzzy system has two variables at input and, each variable is described by 3 membership functions. In order to extract the estimated function $\hat{F}$ and $\hat{G}$ from the estimated function $\hat{f}(\cdot)$, this one is then put under the form:

$$\hat{f}(\Omega,\dot{\Omega}) = \hat{F}(\Omega)\dot{\Omega} + \hat{G}(\Omega). \tag{51}$$

Consequently, the control law i_{qref} is given by:

$$i_{qref} = \hat{F}(\Omega)(v + u_{gl}) + \hat{G}(\Omega) \tag{52}$$

with

$$v = \dot{\Omega}_{ref} + k_{\Omega}(\Omega_{ref} - \Omega), \tag{53}$$

$$u_{gl} = \text{sign}(\Omega_{ref} - \Omega)|\hat{F}(\Omega)|k_{gl}, \quad k_{gl} > 0. \tag{54}$$

5.3. Position control

The procedure used previously is renewed in the case of the tracking of trajectory position. Thus, we again consider the equation (47) where $\dot{\Omega}$ is replaced by position β, which leads to:

$$i_q = F(\dot{\beta})\ddot{\beta} + G(\dot{\beta}) \tag{55}$$

with

$$\beta = \frac{d\Omega}{dt}. \tag{56}$$

To easily derive the control law, the right member of equation (55) is written in the appropriate form:

$$F(\dot{\beta})\ddot{\beta} + G(\dot{\beta}) = f(\dot{\beta},\ddot{\beta}). \tag{57}$$

It is assumed that the function $f(\cdot)$ is perfectly approximated by a fuzzy system such as:

$$f(\dot{\beta},\ddot{\beta}) = W_f(\dot{\beta},\ddot{\beta})\theta_f + \varepsilon_f \tag{58}$$

where θ_f are the optimal parameters. Beside, the estimate $\hat{f}(\cdot)$ of the function $f(\cdot)$ is really calculated by the following expression

$$\hat{f}(\dot{\beta},\ddot{\beta}) = W_f(\dot{\beta},\ddot{\beta})\hat{\theta}_f. \tag{59}$$

This fuzzy system is characterized by two variables $(\dot{\beta},\ddot{\beta})$ at its input. Each variable is described by 3 membership functions regularly distributed on their universe of discourse. In order to derive the estimated function $\hat{F}$ and $\hat{G}$ from the function $\hat{f}(\cdot)$, this one is expressed as follows:

$$\hat{f}(\dot{\beta},\ddot{\beta}) = \hat{F}(\dot{\beta})\ddot{\beta} + \hat{G}(\dot{\beta}). \tag{60}$$

By using the functions ($\hat{F}$ and $\hat{G}$) from the fuzzy system model (60) and in accordance with the control law consequently, the control law i_{qref} is:

$$i_{qref} = \hat{F}(\dot{\beta})(v + u_{gl}) + \hat{G}(\dot{\beta}) \tag{61}$$

where v and the sliding term u_{gl} are given by:

$$v = \ddot{\beta}_{ref} + k_{1\beta}(\dot{\beta}_{ref} - \dot{\beta}) + k_{2\beta}(\beta_{ref} - \beta), \quad (62)$$

$$u_{gl} = \text{sign}((\dot{\beta}_{ref} + \dot{\beta}) + \lambda(\beta_{ref} - \beta))|\hat{F}^{-1}(\dot{\beta})|k_{gl}. \quad (63)$$

5.4. Simulation results

The motor under tests is characterized by: $L_d = L_q = 0.0121H$, $\Phi_f = 0.013Wb$, $j = 0.0001kgm^2$, $F = 0.00005km^2/s$, $R_s = 3.4\Omega$, $\Omega_n = 300rad/s$. The current-controlled inverter is fed by 70V voltage assumed constant. The proposed schema of the adaptive fuzzy controller is tested in simulation to perform the position and speed tracking of PMSM. The values of the control coefficients, which allowed us to obtain satisfactory results, are collected in table 1.

k_Ω	$k_{gl\Omega}$	Γ	$k_{gl\theta}$	λ	$k_{1\theta}$	$k_{2\theta}$
10.852	10.5	10.5	10.5	45	121.7	3702.72

Tab. 1 Control coefficients

The desired trajectories are imposed as:

$$\Omega_{ref} = 300\sin\left(\frac{\pi}{2}t\right), \quad (64)$$

$$\beta_{ref} = \frac{\pi}{2}\left(1 - e^{-0.1t^3}\right)\sin\left(\frac{\pi}{5}t\right). \quad (65)$$

The figures 3 and 4 give respectively the responses of the speed and position control in the case where the nominal load torque is applied. It appears that, the speed and position follow respectively their reference, the disturbance rejection is fast and the stator current is aligned on the q axis (i.e. $i_d = 0$).

The robustness of the trajectories tracking of speed and position is carried out in the presence of the electric parameters variations. Indeed, these variations impose an increase of 100% of the stator resistances, a reduction of 50% of stator inductances and a reduction of 10% of the inductor flux. The obtained responses are represented in figures 5 and 6.

In-spite of the application of these strong parameters variations, at the same time, the tracking speed and position are maintained with a weak tracking error. This shows clearly that this fuzzy adaptive control has the capability to respond quickly to the evolution of the parameters and to their variations.

6. Conclusion

In this work, the adaptive controller based on fuzzy systems was investigated. In this controller, the direct adaptive fuzzy control law ensures the convergence of the tracking

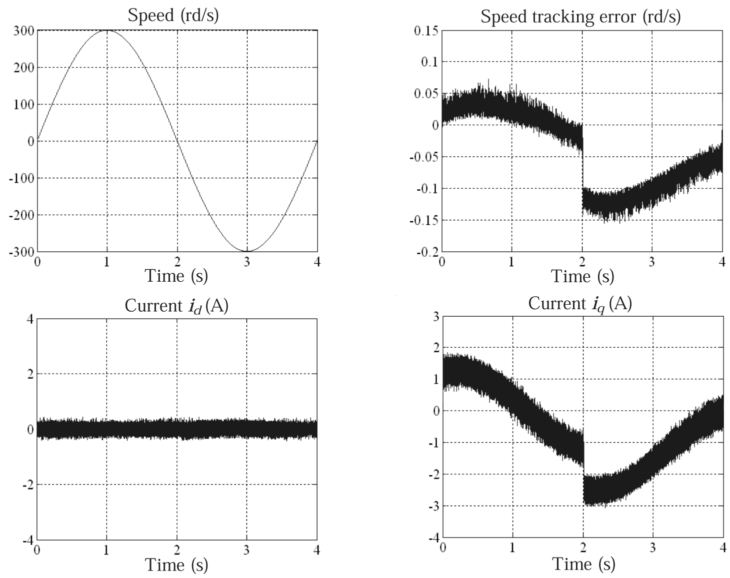

Figure 3. Speed tracking, with the nominal load torque is applied at $t = 2$s, of PMSM

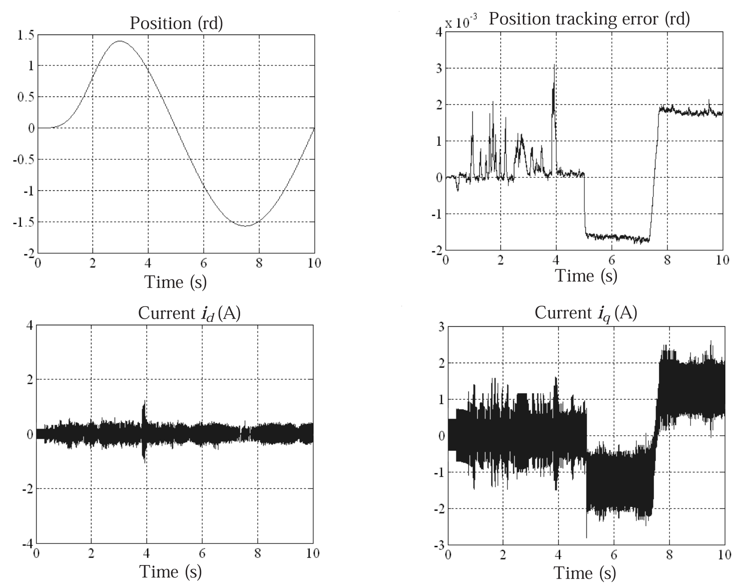

Figure 4. Position tracking, with the nominal load torque is applied at $t = 5$s, of PMSM

errors and boundedness of the fuzzy logic system parameters and all plant signals. This law incorporates an adaptive sliding term to compensate the unknown minimum approximation error between the fuzzy logic model and the controlled plant. This compensation is performed independently of internal or external disturbances. The application of the

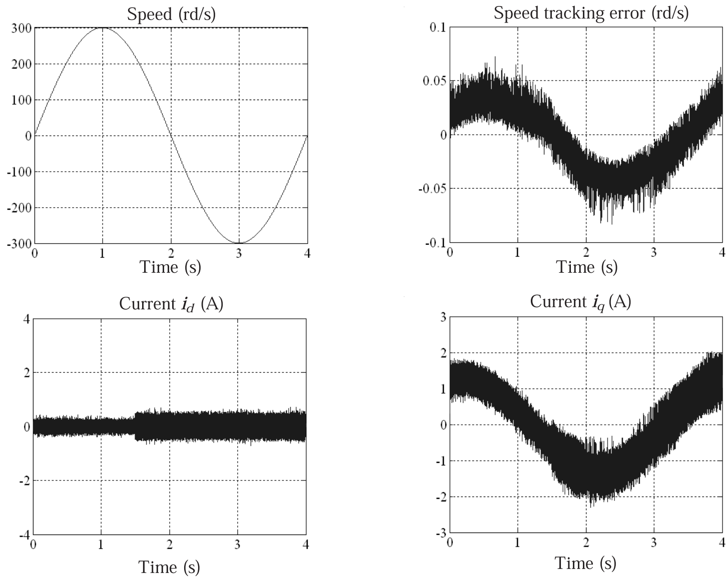

Figure 5. Speed tracking, with parameter variations at $t = 1.5$s, of PMSM

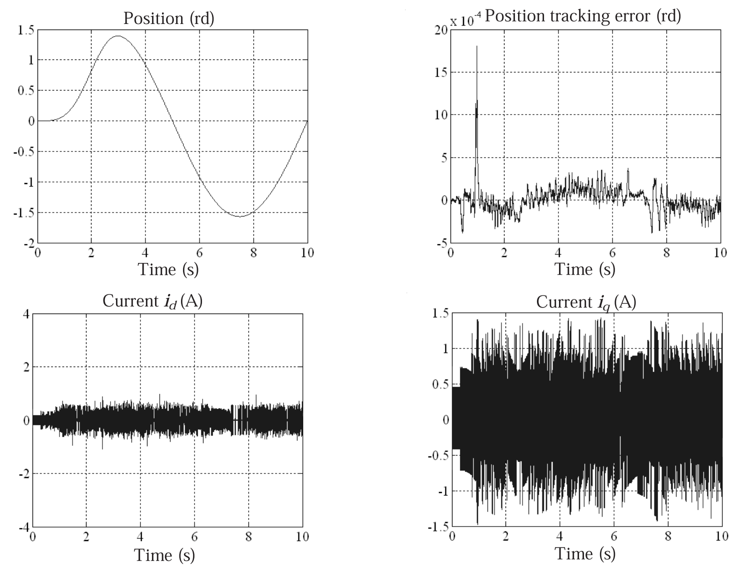

Figure 6. Position tracking, with parameter variations at $t = 1$s, of PMSM

developed method is carried out for a permanent magnet synchronous motor. The obtained simulation results shows that this direct adaptive control fuzzy law maintains the tracking errors in an acceptable interval with feasible control inputs in the presence of hard parameters variations or external disturbances.

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