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Windup prevention in the presence of amplitude and rate saturation

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The purpose of this contribution is to show, that by inserting an appropriate nonlinear dynamic model at the output of the compensator, basically all known techniques for windup prevention developed for amplitude restrictions are also applicable in the presence of actuators with joint amplitude and rate saturation.

Key words: constrained systems, windup prevention, rate saturation

1. Introduction

Due to technological limitations, the output of the compensator cannot be transferred to the system arbitrarily fast and with unlimited amplitudes. If the amplitude and the rate constraints of actuators are not taken into account, both can have a destabilizing effect, the well-known windup. Especially the prevention of windup due to amplitude saturation has been investigated thoroughly (see among others [4], [6], and [14] and the references therein).

As demonstrated in [11] the windup effects can be attributed to two different causes, namely to a mismatch of the compensator states (controller windup) and to a mismatch of the plant states (plant windup). Controller windup is systematically removed by the "Observer Technique" [11], so that the remaining undesired effects of input saturation are then attributable to plant windup. But also the Conditioning Technique [7], the generalized anti windup control [1], or the "Unified Framework" containing all these methods as special cases [12] prevent the windup effects caused by badly damped or unstable modes of the compensator. All these methods assure a stable compensator in case of input saturation, and they do not require additional dynamics.

If one cannot completely remove the undesired effects of input saturation by these methods, i.e., if there exists a plant windup (also called "Short sightedness of the Conditioning Technique" in [7]), additional dynamics have to be introduced like the "Filtered setpoint" [13] or the "Additional Dynamic Network" [11]. Thus one has a two-step approach. First use static measures to stabilize the compensator in case of input saturation,

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and if this does not completely remove all problems caused by the input saturation, introduce additional dynamics to shape the linear part of the loop in an appropriate manner.

A one-step scheme which removes controller and plant windup at the same time was presented by [15]. This scheme incorporates a plant model ("additional dynamics") even if the undesired effects of input saturation can solely be attributed to a controller windup.

The methods mentioned so far apply to open loop stable systems. A solution for the case of exponentially unstable systems with input saturation was presented in [8]. It uses a nonlinear, model based, reference shaping filter for reference tracking and a feedback control for disturbance rejection such that the limited amplitude range of the input signals can be split into one part which is used for disturbance rejection, and the remaining part for tracking. Thus stability can be proven even for exponentially unstable constrained systems which are subject to jointly acting reference and disturbance inputs.

Also the case of joint amplitude and rate constraints has been investigated intensively (see e.g., [5], [3] or [2] and the discussions of the existing approaches therein). The solutions presented so far use an actuator model incorporating two nonlinear elements (for amplitude and rate saturation). Consequently the handling of the problem becomes much more involved as compared to the well-known approaches to the prevention of windup for amplitude restricted actuators.

In this contribution it is shown that basically all known methods for the prevention of windup are readily applicable to the case of actuators with amplitude and rate constraints if a simple ersatz model of such actuators is used in the closed loop design. This model consists of an input saturation and a first order system, whose time constant is chosen such, that the rate constraint is automatically met. By adding the dynamic element to the plant, the problem of windup prevention for amplitude and rate constrained actuators boils down to the well-known problem of amplitude saturation for the augmented plant. A simple example demonstrates the design procedure.

2. Problem formulation

Considered is a linear, time-invariant SIMO system having state $x \in \Re^n$, control input $u_s \in \Re$, controlled output $y \in \Re$, measured output $y_m \in \Re^p$, and a completely controllable and observable state space representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu_s(t), \\ y(t) &= c^T x(t), \\ y_m(t) &= C_m x(t). \end{aligned} \tag{1}$$

There is a saturating nonlinearity $u_s = \mathrm{sat}_{u_0}(u_C) = \mathrm{sign}(u_C)\min\{u_0, |u_C|\}$, $u_0 > 0$ at its input. The transfer function of the linear system (1) is

$$G(s) = c^T(sI - A)^{-1}b = \frac{N(s)}{D(s)} \tag{2}$$

and in view of tracking constant reference signals, it is assumed that $N(s)$ does not contain a zero at $s = 0$. For simplicity assume further that there is a constant state feedback control

$$u_C(t) = -k^T x(t) + mr(t) \tag{3}$$

where $r \in \Re$ is the reference input. Even if the compensator contained a state observer and signal models for disturbance rejection, measures can be taken to remove the so called "controller windup" such that the possibly remaining undesired effects of input saturation, namely the "plant windup", are solely influenced by the feedback vector k^T [9]. It is well known that

$$m = \frac{1}{c^T(-A + bk^T)^{-1}b} \tag{4}$$

assures vanishing steady state errors $y(\infty) - r(\infty)$ for constant reference signals.

If the frequency response $G_L(j\omega)$, where $G_L(s)$ has the form $G_L(s) = k^T(sI - A)^{-1}b$, stays right of the vertical line through $(-1, j0)$ in the complex plane (i.e., $G_L(s)$ meets the circle criterion), the nonlinear loop consisting of an open loop stable linear plant (1), an input nonlinearity $u_s = \text{sat}_{u_0}(u_C)$, and a controller (3) is globally asymptotically stable. If $G_L(s)$ violates the circle criterion, there is the danger of "plant windup", which can be prevented by adding a dynamic element to the loop. This additional dynamic element (ADE) is characterized by

$$u_C(t) = -k^T x(t) - \eta(t) + mr(t) \tag{5}$$

with

$$\begin{aligned} \dot{\xi}(t) &= (A - bk_s^T)\xi(t) + b[u_C(t) - u_s(t)], \\ \eta(t) &= (k^T - k_s^T)\xi(t). \end{aligned} \tag{6}$$

If the "safe" feedback vector k_s^T is chosen such, that $k_s^T(sI - A)^{-1}b$ meets the circle criterion, the nonlinear closed loop is again globally asymptotically stable [9].

The problem addressed now is that the plant input signal u_s differs from the compensator output signal not only whenever

$$|u_C| \geqslant u_0 > 0, \quad \text{(amplitude saturation)} \tag{7}$$

but also whenever

$$|\dot{u}_C| \geqslant u_V > 0, \quad \text{(rate saturation)} \tag{8}$$

i.e., there is an actuator with combined rate and amplitude saturation at the plant input. In [2] it has been suggested to model such actuators by

$$\dot{u}_s(t) = u_V \text{sign}(\text{sat}_{u_0}(u_C(t)) - u_s(t)) \tag{9}$$

where $\text{sat}_{u_0}(\cdot)$ is the above defined saturation function and $\text{sign}(\cdot)$ is the standard sign function. In numerical simulations, however, this model of an amplitude and rate restricted actuator causes problems whenever the argument of the sign function vanishes.

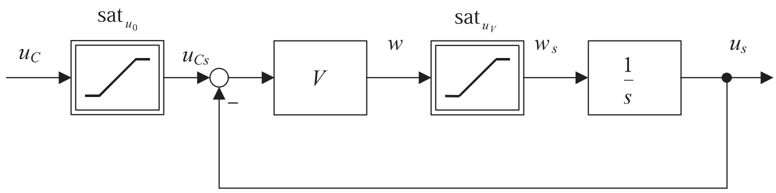

Figure 1. Model of an actuator with combined amplitude and rate saturation

A simple continuous-time model of such actuators which does not cause problems in simulations is shown in Fig. 1 The gain V should be chosen such that the time constant resulting from the integrator and the feedback V does not have significant influence on the behavior of the closed loop. The nonlinear elements in Fig. 1 are of the saturation type, namely

$$u_{Cs} = \mathrm{sat}_{u_0}(u_C) = \mathrm{sign}(u_C)\min\{u_0, |u_C|\}, \quad u_0 > 0 \tag{10}$$

and

$$w_s = \mathrm{sat}_{u_V}(w) = \mathrm{sign}(w)\min\{u_V, |w|\}, \quad u_V > 0. \tag{11}$$

To allow a more compact block diagram representation of the system in Fig. 1, the symbol shown in Fig. 2 is used to depict an actuator with amplitude and rate limitations in the sequel. It is well known that in a closed loop, where the undesired effects of

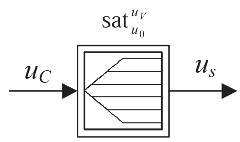

Figure 2. Block diagram symbol for an actuator with amplitude and rate saturation

amplitude saturation have been removed by appropriate measures, additional rate saturation can have a destabilizing influence.

Example 1. Consider a simple third order system (1) with parameters

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}; \; b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}; \; c^T = \begin{bmatrix} -2 & -1 & 1 \end{bmatrix}; \; C_m = \begin{bmatrix} -2 & -1 & 1 \\ 1 & 0 & 0 \end{bmatrix};$$

and consequently $N(s) = 1$ and $D(s) = (s+1)^3$. Due to the static gain of the system and the saturation limit at $u_0 = 2$ the maximum achievable output amplitude is $y_{max} = 2$.

If a constant state feedback with $k^T = [-1701 \;\; -972 \;\; 999]$ and $m = 1000$ is chosen (this k^T places the eigenvalues of the closed loop to $s = -10$), $G_L(s) = k^T(sI-A)^{-1}b$ violates the circle criterion, and if no measures for the prevention of plant windup are taken, the closed loop exhibits limit cycles for sufficiently large reference inputs. Since the transfer function $k_s^T(sI-A)^{-1}b$ satisfies the circle criterion for $k_s^T = [-28 \;\; -20 \;\; 26]$, the nonlinear loop is stable when a dynamic element (6) with this k_s^T is added. The full lines in Fig. 3 show the reference transients of the closed loop, consisting of the plant (1), the feedback (5), and the additional dynamics (6), when a step-like input $r(t) = r_s 1(t) - r_s 1(t-20)$ with $r_s = 1$ is applied. In spite of input saturation, the transients are well damped.

If, however, the saturation element $u_s = \mathrm{sat}_{u_0}(u_C)$ at the input of the system is substituted by an element $u_s = \mathrm{sat}_{u_0}^{u_V}(u_C)$ with combined amplitude and rate saturation $u_0 = 2$ and $u_V = 2$ (see Figs. 1 and 2), one obtains the transients shown in broken lines in Fig. 3. Due to the additional rate limitation $|\dot{u}_C| \leqslant u_V$ they exhibit an oscillatory behavior. To

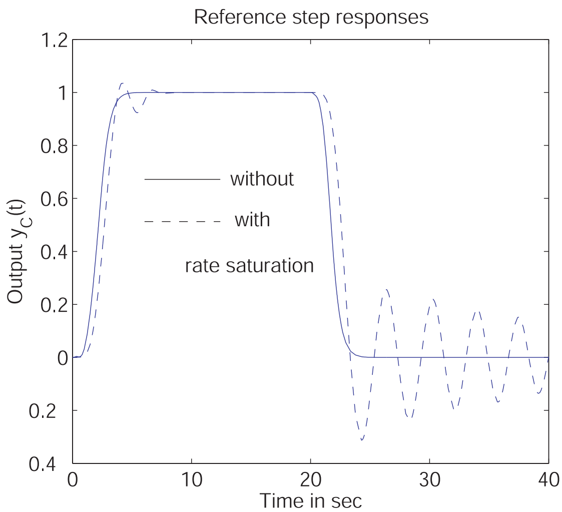

Figure 3. Reference step responses of the loop without and with rate saturation

obtain guaranteed stability of the nonlinear loop, such limitations need to be taken into account when designing a scheme for the prevention of windup.

If one inserted an element according to the Fig. 1 in front of the plant, neither the amplitude nor the rate saturation in the actual actuator would become active, and consequently the design of the closed loop could be carried out for the linear plant augmented by such an actuator model. This, however, would not really facilitate the anti windup design, since the well-known stability criteria for systems with sector nonlinearities apply to linear systems with one isolated nonlinearity. The model in Fig. 1, however, contains two saturation elements. The solution presented in the sequel allows a handling of combined amplitude and rate saturations within the known framework of windup prevention.

3. Windup prevention in the presence joint amplitude and rate saturation

Practically all known measures for the prevention of windup also apply in the case of actuators with combined amplitude and rate saturation, if the ersatz model of Fig. 4 is inserted at the output of the controller. It is obvious that for arbitrary input signals u_C

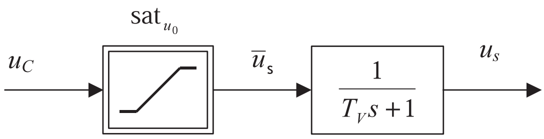

Figure 4. Ersatz model of an actuator with joint rate and amplitude constraints

the output u_s of the model in Fig. 4 never exceeds the amplitude limit u_0. In order to get an output u_s which also satisfies the restriction $|\dot{u}_s(t)| \leqslant u_V$ under all circumstances, the time constant T_V has to be specified as

$$T_V = \frac{2u_0}{u_V} \tag{12}$$

because then, the rate restriction on u_s is also met in the worst case, namely for a sudden change of $\overline{u}_s$ from one saturation limit to the opposite one. When inserting this model at the output of the compensator, neither the amplitude nor the rate saturations in the actual actuator become active. Consequently the actuator nonlinearities do not have to be taken into account. Only the first order system

$$\begin{aligned} \dot{x}_{n+1}(t) &= -\frac{1}{T_V}x_{n+1}(t) + \frac{1}{T_V}\overline{u}_s(t), \\ u_s(t) &= x_{n+1}(t), \end{aligned} \tag{13}$$

needs to be added to the equations (1) of the plant, giving the augmented system

$$\begin{aligned} \dot{\overline{x}}(t) &= \overline{A}\overline{x}(t) + \overline{b}\overline{u}_s(t), \\ y(t) &= \overline{c}^T\overline{x}(t), \\ \overline{y}_m(t) &= \overline{C}_m\overline{x}(t), \end{aligned} \tag{14}$$

with $\overline{x}(t) = \begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix}$, $\overline{y}_m(t) = \begin{bmatrix} y_m(t) \\ u_s(t) \end{bmatrix}$ (since also the additional output $x_{n+1} = u_s$ is measurable) and the parameters

$$\overline{A} = \begin{bmatrix} A & b \\ 0 & \frac{-1}{T_V} \end{bmatrix}, \quad \overline{b} = \begin{bmatrix} 0 \\ \frac{1}{T_V} \end{bmatrix}, \quad \overline{c}^T = \begin{bmatrix} c^T & 0 \end{bmatrix}, \quad \overline{C}_m = \begin{bmatrix} C_m & 0 \\ 0 & 1 \end{bmatrix}.$$

This augmented plant has an input nonlinearity $\overline{u}_s = \mathrm{sat}_{u_0}(u_C)$, so that the considered problem of windup prevention in the presence of combined amplitude and rate constraints boils down to the standard problem of an augmented plant (14) with an amplitude saturated input signal.

Controller windup can, e.g., be removed by the Observer Technique [11]. But also the Conditioning Technique [7] or the generalized scheme presented in [12] are applicable. The possibly remaining plant windup is removable by the above described ADE or by the frequency domain version of this ADE described in [11]. Of course also the one-step scheme for the prevention of controller and plant windup of [15] or the new windup prevention scheme of [8] are applicable. The scheme of [8] uses a feedforward control for tracking and a feedback compensator for stabilization and disturbance rejection. The new scheme in [8] was originally introduced to handle unstable systems with input saturation, but the following examples demonstrate, that there is also an advantage in applying it to stable systems.

Example 2. The proposed anti windup design in the presence of rate saturation is now applied to the system of Example 1. With $u_0 = 2$ and $u_V = 2$ the time constant is $T_V = 2$ (see (12)). With this T_V the equations (14) of the augmented system have the parameters

$$\overline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -2 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -0.5 \end{bmatrix}, \quad \overline{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \end{bmatrix},$$

$$\overline{c}^T = \begin{bmatrix} -2 & -1 & 1 & 0 \end{bmatrix}, \quad \overline{C}_m = \begin{bmatrix} -2 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The state feedback $u_C(t) = -\overline{k}^T\overline{x}(t) + \overline{m}r(t)$ with $\overline{k}^T = [-32076 \;\; -18954 \;\; 19926 \;\; 73]$ and $\overline{m} = 20000$ places all eigenvalues of the closed loop to $s = -10$.

A state observer $\dot{z}(t) = Fz(t) + D\overline{y}_m(t) + T\overline{b}u_C(t)$ of minimal order one with an eigenvalue at $s = -15$ is, e.g., parameterized by $F = -15$, $T = [7 \;\; 7 \;\; -6.5 \;\; 0]$, and $D = [-98 \;\; -98 \;\; 0.5]^T$ and it yields a state estimate

$$\hat{\overline{x}}(t) = \begin{bmatrix} \overline{C} \\ T \end{bmatrix}^{-1} \begin{bmatrix} \overline{y}_m(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 13 & 12 & 0 \\ 14 & 14 & 0 \\ 0 & 0 & 1 \end{bmatrix} \overline{y}_m(t) + \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix} z(t).$$

Applying the one-step scheme of [15] to the problem of windup prevention here, the feedback vector

$$k(x_{aw}) = [-9.22 \;\; -7.76 \;\; 18.02 \;\; 8.5]$$

guarantees global asymptotic stability of the scheme, since $k(x_{aw})(sI - \overline{A})^{-1}\overline{b}$ meets the circle criterion. In spite of the rate saturation $u_V = 2$ the closed loop is now stable. Figure 5 shows (in broken lines) the response of the loop to a reference sequence

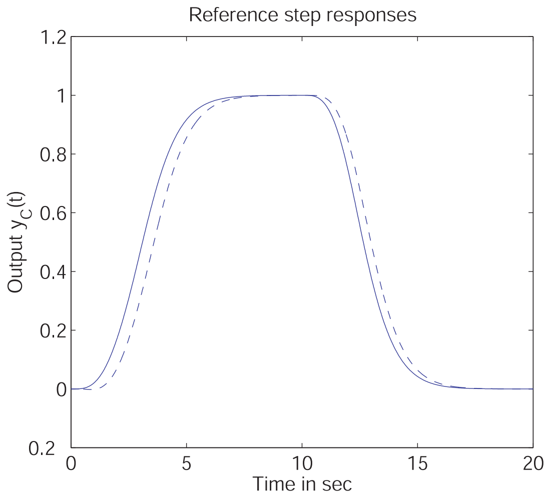

Figure 5. Reference step responses in the presence of rate saturation

$r(t) = r_s 1(t) - r_s 1(t-10)$ with $r_s = 1$. Exactly the same responses of the closed loop would have resulted when the "Observer Technique" for the prevention of controller windup had been applied (i.e., when instead of the unsaturated input signal u_C the saturated input signal $\overline{u}_s$ had been fed into the observer), and when the additional dynamic element according to (5) and (6) with $k_s^T = k(x_{aw})$ had been applied for the prevention of plant windup.

Example 3. An improved behavior of the closed loop can be obtained with the control scheme introduced for unstable systems in [8]. Its structure when applied to the problem investigated here is depicted in Fig. 6. This scheme permits a splitting of the restricted input signal range into one part required for disturbance rejection and the remaining part for reference tracking. This allows a formal proof of stability for exponentially unstable systems. Here in the case of a stable system, the triggering of input saturation by jointly acting reference and disturbance inputs can be allowed, since the stability problems resulting from input saturation can easily be prevented by the additional dynamic element (5), (6). For disturbance rejection, the observer-based compensator considered in Example 2 is applied. The design of the reference filter uses a model

$$\dot{x}_M(t) = A_M x_M(t) + b_M u_{as}(t),$$

$$y_M(t) = c_M^T x_M(t),$$

of the augmented plant (14) and it is assumed, that this model is exact, i.e., one has $(A_M, b_M, c_M^T) = (\overline{A}, \overline{b}, \overline{c}^T)$. For stable systems, the inner stabilizing loop in the filter (see [8]) can be omitted so that for such systems the reference shaping filter with strictly proven stability only consists of one loop with $u_a(t) = -k_a^T x_M(t) + m_a r(t)$ and $u_{as}(t) = \text{sat}_{r_0}(u_a(t))$. It is suggested to use a saturation limit $r_0 < u_0$ in the filter to avoid input

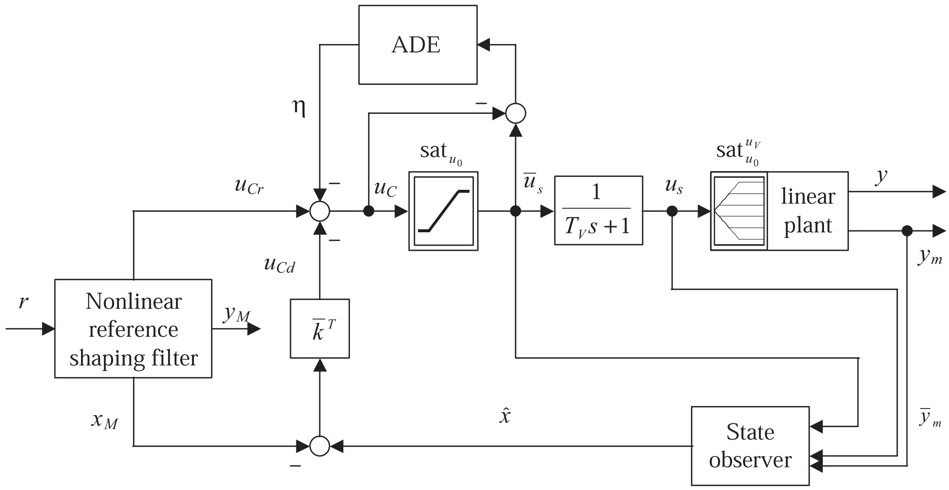

Figure 6. Windup prevention scheme with reference shaping filter

saturation $\overline{u}_s = \text{sat}_{u_0}(u_C)$ due to errors resulting from a finite value of V in the model of Fig. 1. For $V = 200$ problems are avoided by a limit $r_0 = 1.85$. Choosing $k_a^T = k(x_{aw})$ (see Example 2), the reference filter is globally asymptotically stable.

When feeding $u_{Cr}(t) \equiv u_{as}(t)$ into the system in the scheme of Fig. 6, the reference step response shown in the full line in Fig. 5 results. It does not suffer from the (slight) all-pass behavior which delays the transient shown in the broken line. However, for very small changes of the reference signal ($r_s < 1e-04$), the scheme of Example 2 gives linear transients which are characterized by eigenvalues at $s = -10$, whereas the eigenvalues of the (linear) reference shaping filter are located at $s \approx -2$.

As already mentioned in [8], the small-amplitude behavior of the reference filter can be improved considerably by additional cascades. In the sequel, the case of two additional cascades is considered. The modifications necessary for such a multi-loop reference shaping filter are the following. The inner cascade (a) is now characterized by $u_a(t) = -k_a^T x_M(t) + m_a r(t) + u_{bs}(t)$ with $k_a^T = k(x_{aw})$ as above and by $u_{as}(t) = \text{sat}_{r_0}(u_a(t))$. The next cascade (b) is characterized by $u_b(t) = -k_b^T x_M(t) + m_b r(t) + u_{cs}(t)$ and by $u_{bs}(t) = \text{sat}_{r_0}(u_b(t))$, and the outer cascade (c) by $u_c(t) = -k_c^T x_M(t) + m_c r(t)$ and by $u_{cs}(t) = \text{sat}_{r_0}(u_c(t))$. When assigning the feedback vectors k_b^T and k_c^T in the additional cascades as

$$k_b^T = [-752.6444 \quad -516.9202 \quad 637.3402 \quad 18.9] \quad \text{and}$$

$$k_c^T = [-26206.91 \quad -15503.931 \quad 16260.771 \quad 42.4]$$

both transfer functions $k_b^T(sI - \overline{A} + \overline{b}k_a^T)^{-1}\overline{b}$ and $k_c^T(sI - \overline{A} + \overline{b}k_a^T + \overline{b}k_b^T)^{-1}\overline{b}$ satisfy the circle criterion. This, however, only guarantees stability of the filter, if saturation always starts at the outer cascade (c) and progresses inwards, whereas desaturation starts in the inner cascade (a) and progresses outwards.

When choosing

$$m_a = \frac{1}{\bar{c}_C^T(-\bar{A}+\bar{b}k_a^T)^{-1}\bar{b}} = 27.52,$$
$$m_b = \frac{1}{\bar{c}_C^T[-\bar{A}+\bar{b}(k_a^T+k_b^T)]^{-1}\bar{b}} - m_a = 656.2402,$$
$$m_c = \frac{1}{\bar{c}_C^T[-\bar{A}+\bar{b}(k_a^T+k_b^T+k_c^T)]^{-1}\bar{b}} - m_a - m_b = 16303.171, \tag{15}$$

the filter exhibits an oscillatory behavior for reference step inputs in the vicinity of $y_{max} = \frac{N(0)}{D(0)}r_0$, because the saturations and desaturations do not occur in the correct order.

Modifying the factors m_a, m_b and m_c such that the steady-state values of the signals u_a, u_b and u_c coincide, the correct saturation and desaturation orders results. For the system considered here, coinciding steady state values for the signals u_a, u_b and u_c are assured by the modified factors $m_a^* = m_a - 1$, $m_b^* = m_b$ and $m_c^* = m_c + 1$. Though there is no formal proof for it, the results obtained so far seem to indicate, that by an appropriate modification of the factors m_a, m_b and m_c such that the steady-state values of all input signals (here of u_a, u_b and u_c) coincide, stability problems can be prevented for a great variety of applications (for a more comprehensive discussion of this see [10]). Figure 7

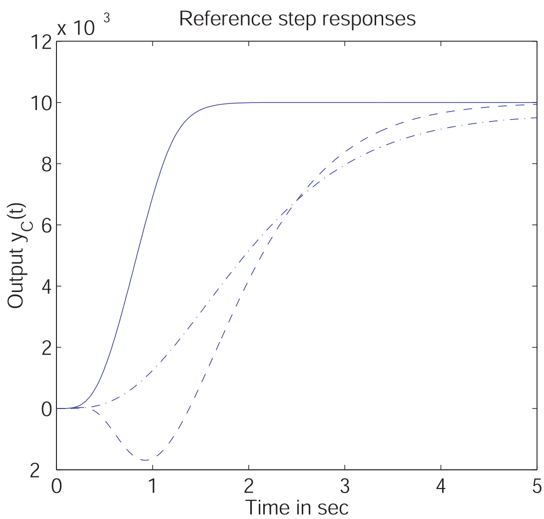

Figure 7. Comparison of the small-amplitude behaviour of the schemes in Examples 2 and 3

shows a comparison of the schemes presented in the Examples 2 and 3 for a reference step amplitude $r_s = 0.01$. The step response in broken lines results with the scheme of Example 2, and the one in dash-dotted lines with the (single loop) reference shaping filter of Example 3. The response in full lines is obtained with the reference shaping filter having three cascades (a), (b) and (c).

The linear behaviour of the three-loop reference filter is about the same as that of the scheme in Example 2, so that for extremely small input amplitudes ($r_s < 1e-04$) the results of the scheme in Example 2 and of the three-loop filter coincide. For all reference signals triggering input saturation, however, the three-loop filter definitely yields much better results. This is a consequence of the fact, that in the schemes of Example 2 input saturation causes a backlash in the signal $\overline{u}_s$, giving rise to an all-pass behavior in the transients (see broken line in Fig. 7). The reference shaping filter does not suffer from this drawback.

4. Summary

It has been demonstrated, that by inserting a special model of the amplitude and rate restricted actuator at the output of the controller, the well-known approaches to the prevention of windup for systems with input amplitude saturation become also applicable to solving the problems caused by additionally rate restricted actuators. This special model consists of an input amplitude saturation and a first order system, whose time constant is chosen such that the rate constraint is satisfied even in worst case situations. The results have been presented for SIMO systems only, since it is obvious, how this approach can be applied in the MIMO case as well.

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