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Self-tuning LQG control with amplitude-constrained input. Robustness aspects

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Self-tuning tracking control of ARMAX system in the presence of amplitude constrained input is considered. An extension of the result presented in [7] is given to the case of piecewise constant set-point. Stability and performance robustness are discussed on the basis of second-order ARMAX systems.

Key words: LQG tracking control, amplitude constraint, self-tuning

1. Introduction

Various approaches have been proposed to solve the self-tuning tracking control problem in the presence of input constraints. For example, this problem was considered in [8], [6] for stochastic ARMAX system, and in [9], [1] for bounded noise ARMAX system. In [7], a suboptimal stochastic control algorithm has been proposed for discrete-time state-space stochastic system subjected to the amplitude-constrained input. The key idea of this algorithm is the approximation (due to the saturation input constraint) of the stationary probability density function (pdf) of the state by the gaussian pdf. The gradient of the cost function is then used to derive the iterative procedure for calculation of the stationary feedback gain. In this paper, the above approach is extended to the LQG indirect self-tuning tracking control problem of stochastic ARMAX system with constant or piecewise constant set-point.

2. Problem formulation

The system is given by the following ARMAX model

$$A(q^{-1})y_t = B(q^{-1})u_t^c + C(q^{-1})e_t \tag{1}$$

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where $A(q^{-1}), B(q^{-1}), C(q^{-1})$ are polynomials in the backward shift operator q^{-1}, i.e.,

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_n q^{-n}$$

$$B(q^{-1}) = b_1 q^{-1} + ... + b_n q^{-n}$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_n q^{-n}$$

and y_t, u_t^c are the system output and the constrained control input, respectively. Moreover, $\{e_t\}$ is assumed to be a sequence of independent gaussian variables with zero mean and variance σ_e^2. An ARMAX model (1) has an equivalent innovation state space representation

$$\underline{x}_{t+1} = F\underline{x}_t + \underline{g}u_t^c + \underline{k}e_t \tag{2}$$

$$y_t = \underline{h}^T \underline{x}_t + e_t \tag{3}$$

where

$$\underline{g} = (b_1, ..., b_n)^T, \qquad \underline{k} = (c_1 - a_1, ..., c_n - a_n)^T$$

$$\underline{h}^T = (1, 0, ..., 0)$$

$$F = \begin{bmatrix} -a_1 & 1 & ... & 0 \\ . & . & ... & 0 \\ -a_{n-1} & . & ... & 1 \\ -a_n & . & ... & 0 \end{bmatrix}$$

and $\underline{k}$ is the stationary gain vector of the associated Kalman filter

$$\hat{\underline{x}}_{t+1} = F\hat{\underline{x}}_t + \underline{g}u_t^c + \underline{k}\widetilde{y}_t \tag{4}$$

where $\widetilde{y}_t = y_t - \underline{h}^T \hat{\underline{x}}_t$.

The cost function for the tracking control problem

$$J = E[(y_t - r_t)^2 + q u_t^{c2}] \tag{5}$$

has to be minimized by a suitable control algorithm under the amplitude-constrained control input implemented by the saturation function

$$u_t^c = sat(u_t; \alpha) \tag{6}$$

where

$$sat[u_t; \alpha] = \begin{cases} +\alpha & \text{for } u_t > +\alpha \\ u_t & \text{for } +\alpha \leqslant u_t \leqslant -\alpha \\ -\alpha & \text{for } u_t < -\alpha \end{cases}$$

and u_t is given by some linear feedback control law.

The known set-point r_t is assumed to be constant or piecewise constant. First, the non-adaptive case is considered when the system parameters $\underline{\theta} = (a_1, ..., a_n, b_1, ..., b_n, c_1, ..., c_n)^T$ are assumed to be known.

3. Control algorithm for zero set-point

The suboptimal amplitude-constrained control algorithm has been derived by Toivonen in [2] for the regulation problem ($r_t = 0$) defined by the cost function

$$J_1 = E[\underline{x}_t^T Q \underline{x}_t + q u_t^{c2}] = trQR_x + q\sigma_u^2 \tag{7}$$

where $R_x = E\underline{x}_t\underline{x}_t^T$. Using any stabilizing feedback control law, the stationary equation for $R_{\hat{x}} = E\hat{\underline{x}}_t\hat{\underline{x}}_t^T$ resulting from (4) is

$$R_{\hat{x}} = FR_{\hat{x}}F^T + FR_{\hat{x}u}\underline{g}^T + \underline{g}R_{\hat{x}u}^T F^T + \underline{g}\sigma_u^2\underline{g}^T + \underline{kk}^T\sigma_e^2 \tag{8}$$

where the approximate expressions for σ_u^2 and $R_{\hat{x}u}$ under the constrained control law

$$u_t^c = sat(\underline{f}^T\hat{\underline{x}}_t;\alpha) \tag{9}$$

are [2]

$$\sigma_u^2 = \sigma^2 g_1(\sigma) \tag{10}$$

$$R_{\hat{x}u} = R_{\hat{x}}\underline{f}g_2(\sigma) \tag{11}$$

and

$$\sigma^2 = \underline{f}^T R_{\hat{x}}\underline{f} \tag{12}$$

$$\begin{aligned} g_1(\sigma) &= erf(\alpha\sigma^{-1}2^{-\frac{1}{2}}) - \\ &\quad -\alpha\sigma^{-1}2^{\frac{1}{2}}ierfc(\alpha\sigma^{-1}2^{-\frac{1}{2}}) \qquad (13) \\ g_2(\sigma) &= erf(\alpha\sigma^{-1}2^{-\frac{1}{2}}). \qquad (14) \end{aligned}$$

Introducing (10)-(14) into (7), Toivonen [2] proposed an iterative method for calculating the feedback gain $\underline{f}$ in the control law (9) which minimizes J_1.

Let the weight matrix Q be given as $Q = \underline{hh}^T$, then from (3) it follows

$$\sigma_y^2 = r_{11} + \sigma_e^2 \tag{15}$$

where r_{11} is the *11* element of R_x, so the minimization of (7) is equivalent to the minimization of (5) if $r_t = 0$.

4. Control algorithm for nonzero set-point

The optimal unconstrained linear control law for nonzero constant or piecewise constant set-point $r_t = r$ is

$$u_t = \underline{f}^{xT}\hat{\underline{x}}_t + f^r r \tag{16}$$

where [4]

$$\underline{f}^{xT} = -(\underline{g}^T P^{11} \underline{g} + q)^{-1} \underline{g}^T P^{11} F \quad (17)$$

$$f^r = -(\underline{g}^T P^{11} \underline{g} + q)^{-1} \underline{g}^T P^{12} \quad (18)$$

and P^{11}, P^{12} follow from Riccati equations

$$P^{11} = F^T P^{11} F - F^T P^{11} \underline{g} (\underline{g}^T P^{11} \underline{g} + q)^{-1} \underline{g}^T P^{11} F + \underline{h} \underline{h}^T \quad (19)$$

$$P^{12} = (F - \underline{g} \underline{f}^{xT})^T P^{12} - \underline{h} \quad (20)$$

or [3] using the transfer function

$$G(z) = \underline{h}^T (zI - F + \underline{g} \underline{f}^{xT})^{-1} \underline{g}, \qquad f^r = G^{-1}(1) \quad (21)$$

where $\underline{f}^x$ is any stabilizing feedback gain, for example it can be calculated from (17). It is worthy to notice, as numerical calculations show, that the gain f^r calculated from (21) for $\underline{f}^x$) (17) is equivalent to that calculated from (18).

Now consider the case of amplitude-constrained control. It can be shown that the method described in Section 3 can be generalized to the nonzero set-point case. To show it, consider eqn.(8) for zero mean variables

$$u_t' = u_t - u, \quad \underline{\hat{x}}_t' = \underline{\hat{x}}_t - \underline{x}$$

where $u = Eu_t = \underline{f}^{xT} \underline{\hat{x}} + f^r r$ and $\underline{\hat{x}} = E\underline{\hat{x}}_t = [I - F - \underline{g} \underline{f}^{xT}]^{-1} \underline{g} f^r r$.
Note that the covariance matrix of $\underline{\hat{x}}_t'$ is

$$R_{\hat{x}}' = R_{\hat{x}} - R_{\underline{\hat{x}}} \quad (22)$$

while the covariance $R_{\hat{x}u}''$ is defined as

$$R_{\hat{x}u}'' = E[\underline{\hat{x}}_t' u_t'] = R_{\hat{x}} \underline{f}^x - R_{\underline{\hat{x}}} \underline{f}^x$$

where $R_{\underline{\hat{x}}} = \underline{\hat{x}} \underline{\hat{x}}^T$, and using (22)

$$R_{\hat{x}u}'' = R_{\hat{x}}' \underline{f}^x. \quad (23)$$

Moreover, the variance $\sigma_{u'}^2$ is given by

$$\sigma_{u'}^2 = E[u_t'^2] = \underline{f}^{xT} R_{\hat{x}} \underline{f}^x - \underline{f}^{xT} R_{\underline{\hat{x}}} \underline{f}^x.$$

Again, using (22) yields

$$\sigma_{u'}^2 = \underline{f}^{xT} R_{\hat{x}}' \underline{f}^x. \quad (24)$$

Correspondingly to (10),(11) one gets

$$\sigma_u^{'2} = \sigma_{u'}^2 g_1(\sigma_{u'}) \tag{25}$$

$$R'_{\hat{x}u} = R''_{\hat{x}u} g_2(\sigma_{u'}) \tag{26}$$

under the saturated control law

$$u_t^c = sat(u'_t; \alpha). \tag{27}$$

Similarly to (8), the following equation can be written

$$R'_{\hat{x}} = FR'_{\hat{x}}F^T + FR'_{\hat{x}u}\underline{g}^T + \underline{g}R'^T_{\hat{x}u}F^T + \underline{g}\sigma_u^{'2}\underline{g}^T + \underline{k}\underline{k}^T\sigma_e^2. \tag{28}$$

Equation (28) can also be written in terms of $R_{\hat{x}}$ and $R_{\underline{x}}$

$$R_{\hat{x}} = FR_{\hat{x}}F^T + g_2(FR_{\hat{x}}\underline{f}^x\underline{g}^T + \underline{g}\underline{f}^{xT}R_{\hat{x}}F^T) +$$
$$+ g_1\underline{g}\underline{f}^{xT}R_{\hat{x}}\underline{f}^x\underline{g}^T + \underline{k}\underline{k}^T\sigma_e^2 + R_{\underline{\hat{x}}} - FR_{\underline{\hat{x}}}F^T -$$
$$- g_2(FR_{\underline{\hat{x}}}\underline{f}^x\underline{g}^T + \underline{g}\underline{f}^{xT}R_{\underline{\hat{x}}}F^T) - g_1\underline{g}\underline{f}^{xT}R_{\underline{\hat{x}}}\underline{f}^x\underline{g}^T. \tag{29}$$

The known set-point r is needed for calculation of $\underline{\hat{x}}$ and $R_{\underline{\hat{x}}}$. Putting $\sigma_u^{'2}$ (25), $R'_{\hat{x}u}$ (26) into (28) and using (23) and (24), an equation for $R'_{\hat{x}}$ can be obtained which in turn can be further used for optimization of $\underline{f}^x$ in the same line as described in Section 3. Obviously, the resulting feedback gain $\underline{f}^x$ will be the same as $\underline{f}$ of the saturated control law (9).

5. Closed-loop stability analysis

Consider the unconstrained control system and the corresponding closed-loop equation which can be obtained by combining equations (2),(3),(4) and (16), i.e., when $u_t^c = u_t$

$$\begin{bmatrix} \underline{x}_{t+1} \\ \underline{\widetilde{x}}_{t+1} \end{bmatrix} = \begin{bmatrix} F + \underline{g}\underline{f}^{xT} & \underline{g}\underline{f}^{xT} \\ 0 & F - \underline{k}\underline{h}^T \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{\widetilde{x}}_t \end{bmatrix} +$$
$$+ \begin{bmatrix} \underline{g}f^r & \underline{k} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ e_t \end{bmatrix} \tag{30}$$

where $\underline{\widetilde{x}}_t = \underline{\hat{x}}_t - \underline{x}_t$ is the filtration error. The roots z_i of the characteristic equation can be found from (30). To give an insight into how the controller attempts to cope with constraint on the control signal consider $\mid z_i \mid_{max} = \mid z \mid_{max}$ calculated from (30) for a linear feedback control law with feedback gain $\underline{f}^x$ obtained from the minimization procedure described in Section 4. To this end, consider the following second-order examples:

1. stable ARMAX system: $a_1 = -1.8, a_2 = 0.9, b_1 = 1.0, b_2 = 0.5$,
2. unstable ARMAX system: $a_1 = 1.8, a_2 = -0.9, b_1 = 1.0, b_2 = 0.5$,
3. non-minimum phase ARMAX system: $a_1 = -1.8, a_2 = 0.9, b_1 = -1.0, b_2 = 2.0$.

The polynomial C was taken as $C = 1$, the noise variance σ_e^2 was set at 0.1, and $q = 0.1$.

The resulting constrained feedback gains, covariance matrices $R_{\hat{x}}$ and the values of $\mid z \mid_{max}$ obtained from equation (30) for different values of constraint α are given in Tables 1,2,3 for examples 1,2,3, respectively, where the set-point $r = 3.0$. For stable and unstable systems one can observe an increase of stability margin measured by $\mid z \mid_{max}$ along with a decrease of constraint α. The nonminimumphase system is rather insensitive in this respect. However, it should not be forgotten that $\mid z \mid_{max}$ is obtained for linear control system but with the feedback gain calculated as for constrained control law. In fact, when the constrained feedback is applied the analysis of closed-loop stability based on eqn.(30) is not valid anymore, i.e. the closed-loop system can be unstable in spite of stable pole $\mid z \mid_{max}$. This can happen for unstable open-loop systems for which one can not assure the global closed-loop stability, i.e. for any constraint α.

Table 1. Optimization results for example 1

α	f^x	f^r	$R_{\hat{x}}$		$\|z\|_{max}$
0.8	-1.237 -0.829	0.586	9.457 -8.223	-8.223 7.433	0.164
1.0	-1.304 -0.843	0.639	9.417 -8.180	-8.180 7.435	0.184
2.0	-1.439 -0.872	0.751	9.356 -8.102	-8.102 7.483	0.225
3.0	1.450 -0.874	0.759	9.353 -8.097	-8.097 7.490	0.227
∞	-1.450 -0.874	0.759	9.353 -8.097	-8.097 7.490	0.227

Table 2. Optimization results for example 2

α	f^x	f^r	$R_{\hat{x}}$		$\|z\|_{max}$
5.0	1.974 -1.017	0.852	9.337 13.756	13.756 22.980	0.457
7.0	1.952 -1.010	0.864	9.332 13.746	13.746 22.958	0.475
8.0	1.952 -1.010	0.864	9.332 13.745	13.745 22.906	0.475
∞	1.952 -1.010	0.864	9.332 13.745	13.745 22.906	0.475

Table 3. Optimization results for example 3

α	f^x	f^r	$R_{\hat{x}}$		$\lvert z\rvert_{max}$
0.8	-1.247 -1.250	0.472	10.999 -8.327	-8.327 6.569	0.509
1.0	-1.264 -1.262	0.481	10.990 -8.332	-8.332 6.589	0.499
2.0	-1.271 -1.267	0.484	10.987 -8.335	-8.335 6.600	0.494
3.0	-1.271 -1.267	0.484	10.987 -8.335	-8.335 6.600	0.494
∞	-1.271 -1.267	0.484	10.987 -8.335	-8.335 6.600	0.494

Table 4. Sample costs

Example	α	$\bar{J}$	σ_u^2
LQG-1	1.0	3.959	0.320
LQG-1	3.0	2.762	0.628
LQG-1	5.0	2.491	0.495
LQG-2	8.0	4.262	32.383
LQG-2	9.0	4.160	31.453
LQG-2	10.0	4.032	28.069
LQG-3	1.0	10.266	0.434
LQG-3	3.0	10.177	0.495
LQG-3	5.0	12.973	0.807

6. Self-tuning control: second order systems

In the self-tuning implementation, the system parameter estimates $\underline{\hat{\theta}}$ are obtained on-line via RELS method, and the updated model is used for the control of the system.

Stability of self-tuning control system under amplitude-constrained input can be viewed separately for stable and unstable systems. It is well known that for unstable systems it is not possible to assure global closed-loop stability for LQG control system even in nonadaptive case. However, in the case of noise-free (or bounded-noise systems), an adaptive controller with amplitude-constrained output may locally stabilize the system. Then some closed-loop stability-instability boundary can be evaluated in terms of initial parameter uncertainty and initial conditions or magnitude of the reference. This will be discussed in next Section.

In the nonadaptive case, i.e.when model parameters are known the problem is related to the determination of domain of attraction for saturated control system.

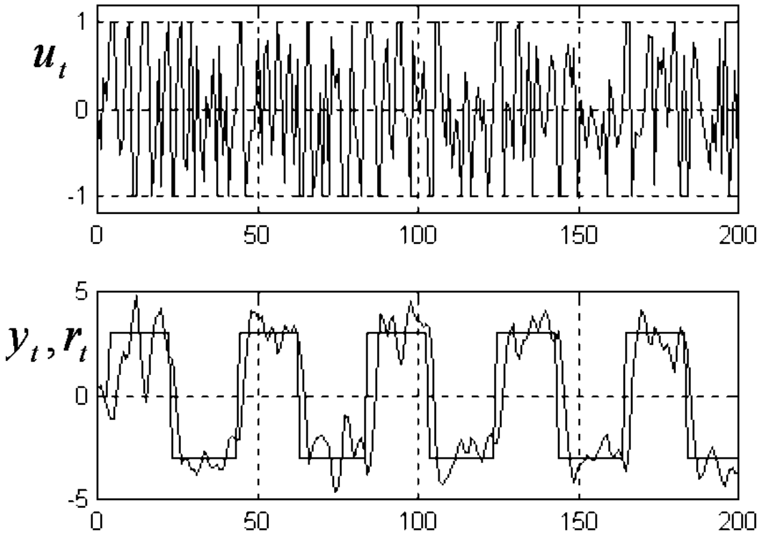

Figure 1. Control behavior for $\alpha = 1$, example 1

In the following simulations, the initial estimates are taken as $\hat{a}_1 = -1.6, \hat{a}_2 = 0.7, \hat{b}_1 = 0.9, \hat{b}_2 = 0.6$ for example 1, $\hat{a}_1 = 1.6, \hat{a}_2 = -0.7, \hat{b}_1 = 0.9, \hat{b}_2 = 0.6$ for example 2 and $\hat{a}_1 = -1.6, \hat{a}_2 = 0.7, \hat{b}_1 = -0.9, \hat{b}_2 = 1.6$ for example 3. Selected simulation runs of examples 1,2,3 are presented for $q = 0.1, \sigma_e^2 = 0.1$, where the set-point was taken as

$$r_{20N+t+5} = 3(-1)^N, \quad t = 0, \cdots 49, \qquad N = 0, 1, \cdots$$

The results of self-tuning constrained LQG control for example 1 are shown in Fig.1 for constraint $\alpha = 1$. The tracking accuracy deteriorates significantly when $\alpha < 1$. The stable realization of response shown in Fig.2 is obtained for $\alpha = 8$ in the case of example 2. Obviously, this unstable open-loop system can not generally be stabilizable in arbitrary constrained self-tuning context. Besides, it is known that even in nonadaptive case there is a positive probability that $\{\underline{x}_t\}$ will drift to infinity and never come back in the presence of input constraint. In Fig.3, the control behavior for example 3 is shown with $\alpha = 1$.

It is obvious that the tracking performance under soft constraints is always better. The averaged values of the sample cost function $\bar{J}$ for examples 1,2,3 are given in Table 4, where the results obtained for example 2 concern obviously the stable control realizations.

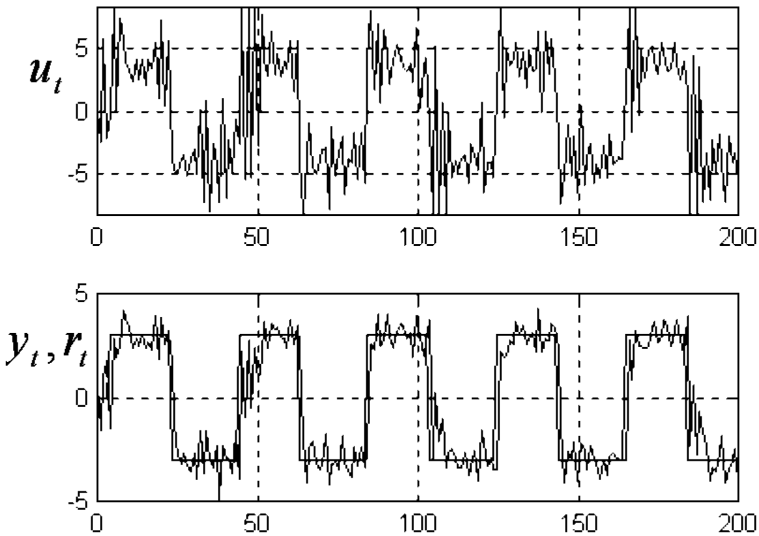

Figure 2. Control behavior for $\alpha = 8$, example 2

7. Robustness aspects

Two general aspects of robustness can be considered: robust stability and robust performance.

7.1. Robust stability

First, consider the stability of unconstrained LQG control with uncertainty in gain parameters $\underline{g}$ given by the perturbation

$$\underline{g}_\mu = \mu \underline{g} \tag{31}$$

where $\mu > 0$, for the control law given for simplicity by

$$u_t = \underline{f}^{xT} \underline{x}_t + f^r r \tag{32}$$

i.e., with perfect state measurement.

It can be shown [5] that for a given weight q, the lower and upper limit for μ ensuring the closed-loop stability of the perturbed system (31) is given by

$$\mu^s_{min} = (1+a)^{-1}, \tag{33}$$

$$\mu^s_{max} = (1-a)^{-1} \tag{34}$$

where

$$a = \sqrt{\frac{q}{q + \underline{g}^T P^{11} \underline{g}}} \tag{35}$$

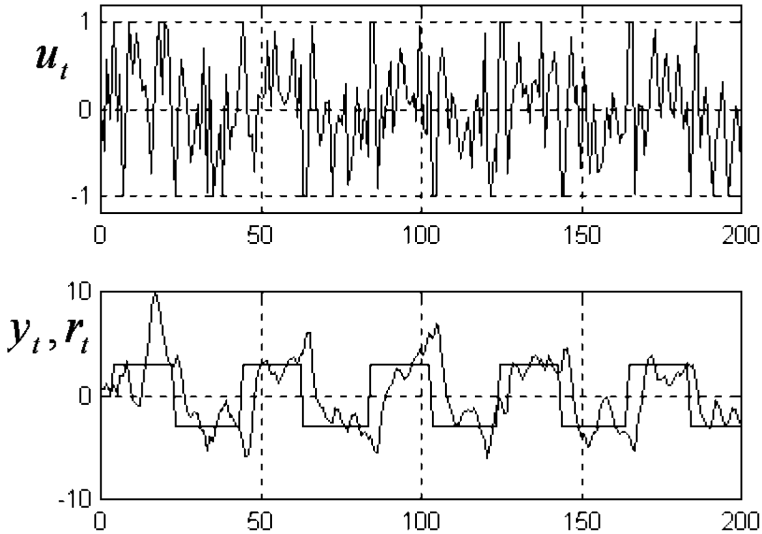

Figure 3. Control behavior for $\alpha = 1$, example 3

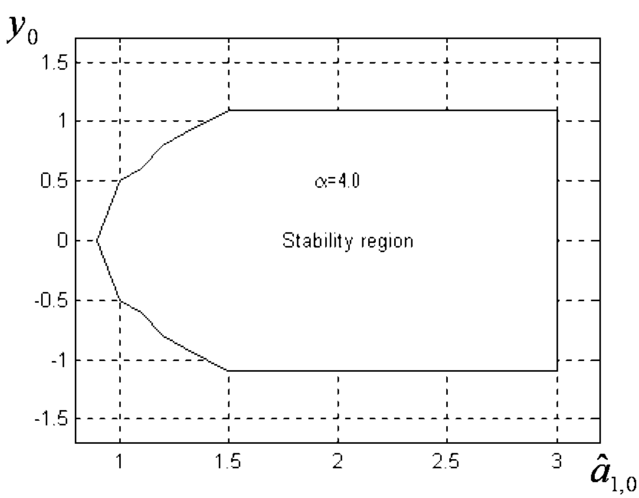

Figure 4. Stability region

and the matrix P^{11} is the solution of the Riccati equation (19) for the nominal system. The second Riccati equation (20) is not crucial at this moment because the closed-loop stability is not influenced by the set-point.

Successively, the parametric uncertainty in matrix F given by the perturbation

$$F_{\mu} = F(\mu \underline{a}) \tag{36}$$

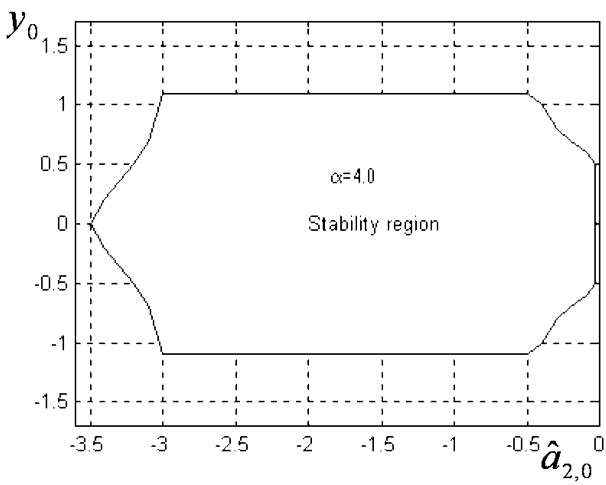

Figure 5. Stability region

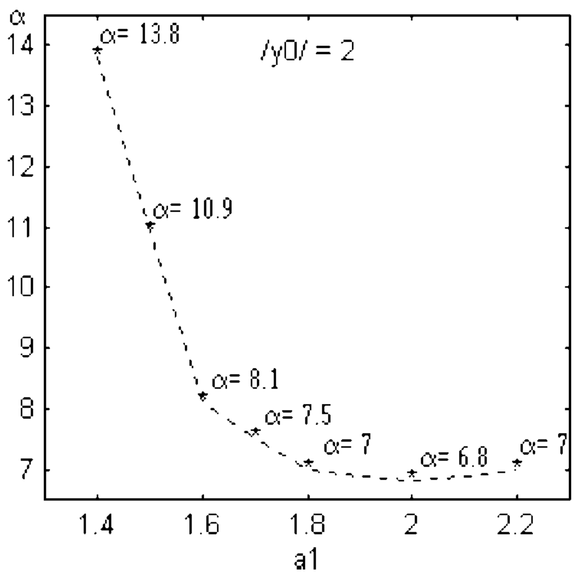

Figure 6. Stability robustness limits

is considered where $\underline{a} = (a_1, \cdots, a_n)$. The problem of finding μ^s_{min}, μ^s_{max} is related to the determination of stability radius of the stable matrix $F + \underline{gf}^{xT}$ with respect to perturbation μ. No analytical results like (33),(34) are available, however the bounds on μ can be found numerically by checking the condition $\mid z \mid_{max} < 1$ from the characteristic equation of the perturbed matrix $F(\mu\underline{a}) + \underline{gf}^{xT}$, i.e.

$$det(zI - F(\mu\underline{a}) - \underline{gf}^{xT}) = 0 \qquad (37)$$

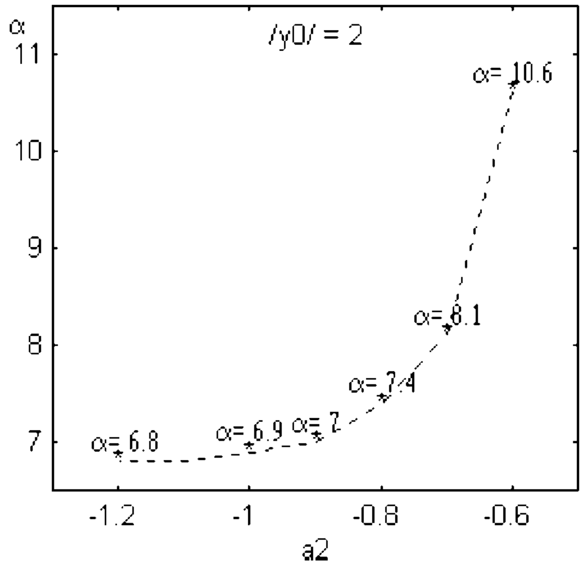

Figure 7. Stability robustness limits

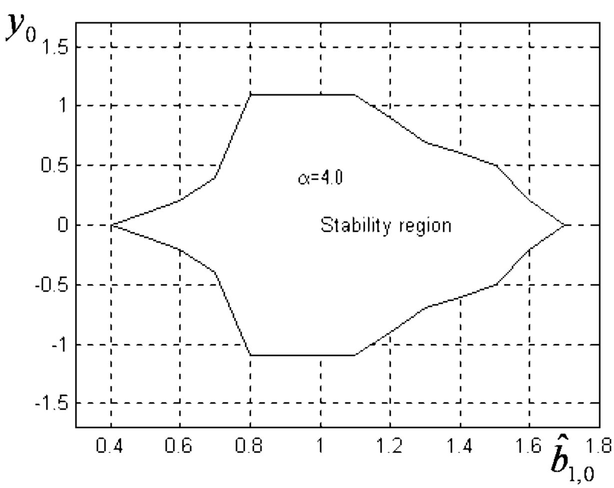

Figure 8. Stability region

where $\underline{f}^x$ is calculated from (17) with P^{11} obtained from (19) for the nominal system.

The similar robustness analysis can be performed with respect to the stability of the Kalman filter, see eqn.(30), which yields the perturbed characteristic equation

$$det(zI - F(\mu\underline{a}) + \underline{kh}^T) = 0. \tag{38}$$

To illustrate the above consider the second-order stable and unstable systems, i.e., examples 1, 2, respectively, and $q = 0.1$. For perturbation (31), the bounds (33), (34) for

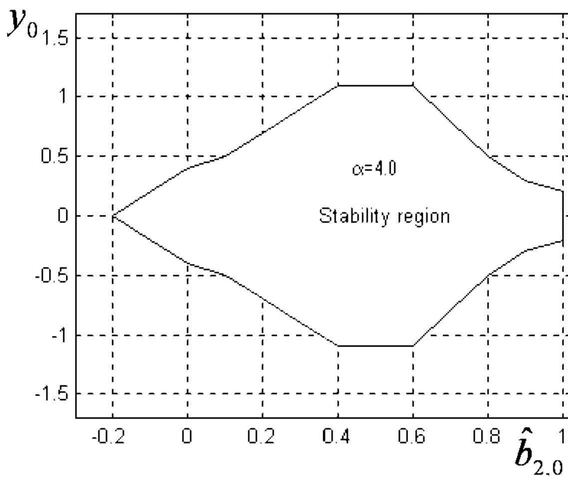

Figure 9. Stability region

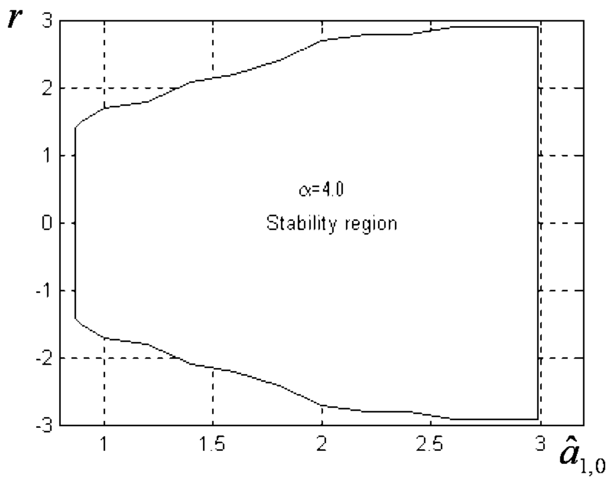

Figure 10. Stability region

example 1 are $\mu^s_{min} = 0.81, \mu^s_{max} = 1.32$. The corresponding values for example 2 are $\mu^s_{min} = 0.79, \mu^s_{max} = 1.34$.

For perturbation (36), the corresponding results are $\mu^s_{min} = 0.15, \mu^s_{max} = 1.45$ for example 1, and $\mu^s_{min} = 0.55, \mu^s_{max} = 1.65$ for example 2 when the characteristic equation (37) is considered. In the case of characteristic equation (38), the limits are $\mu^s_{min} = 0.65, \mu^s_{max} = 2.10$ for example 1, and $\mu^s_{min} = -0.22, \mu^s_{max} = 1.35$ for example 2.

Now, consider the robust stability of constrained adaptive control system. In this case, the stability robustness can be investigated taking into account the uncertainty

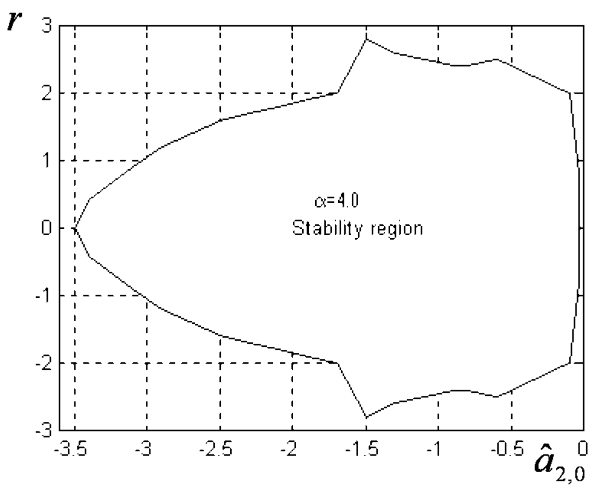

Figure 11. Stability region

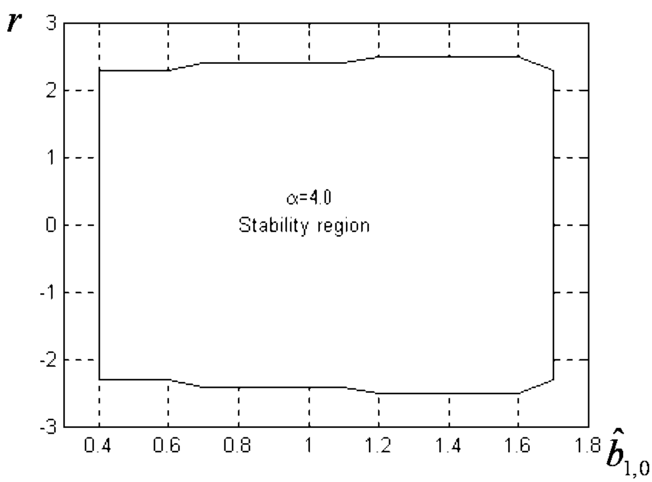

Figure 12. Stability region

about the system parameters. For unstable noise-free systems, it is possible to evaluate the closed-loop stability-instability boundaries with respect to amplitude constraint α in terms of initial conditions and parametric uncertainty. To this end consider first the second-order unstable system (example 2) for the regulation problem, i.e. $r_t = 0$. Stability boundaries depending on the initial condition y_0 and initial parameter estimates $\hat{a}_1(0)$ and $\hat{a}_2(0)$ are depicted in Fig.4 and Fig.5, respectively for the constraint $\alpha = 4$. Stability regions are inside the $\alpha = 4$ contour. Plots of destabilizing values of α versus uncertain parameters a_1 and a_2 taken for the controller design with the initial condition

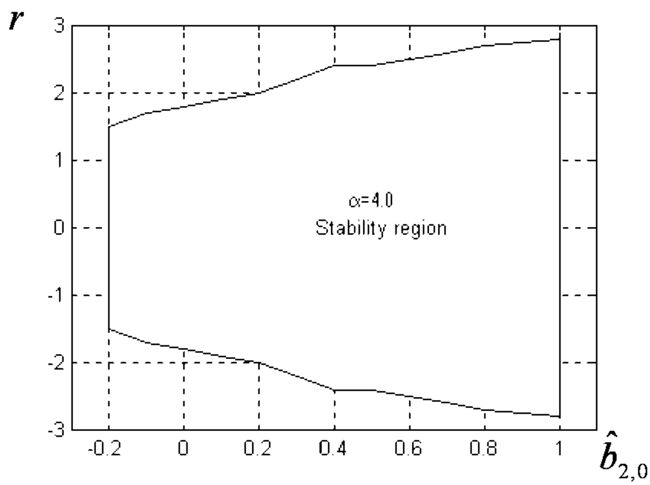

Figure 13. Stability region

$| y_0 |= 2$, are depicted in Figs.6,7, respectively. It is interesting to see that the lower allowable value of constraint $\alpha \approx 6.8$ occurs close to actual values of parameters a_1, a_2.

Similar stability-instability boundary can be obtained with respect to uncertain parameters b_1 and b_2. The corresponding stability regions are shown in Figs.8,9 for $\alpha = 4$.

Analogous stability analysis can be done with respect to the magnitude of the set-point r. This case is illustrated in Figs.10,11,12,13 for $\alpha = 4$ and zero initial condition $y_0 = 0$.

7.2. Robust performance

First, it is worthy to note that in order to assure the tracking condition of constrained control system, the constant set-point r must fulfill

$$| r |< \frac{| B(1) |}{| A(1) |}\alpha. \tag{39}$$

As pointed out in Section 6, the tracking accuracy can be significantly deteriorated in the self-tuning implementation for some α although the condition (39) is fulfilled . To preserve a robust performance in self-tuning control system can be difficult even in unconstrained case. As shown in [5], the only way that the self-tuning control law can asymptotically converge to the optimal control law (16) is for the parameter estimates$\hat{\underline{\theta}}$ to converge precisely to $\underline{\theta}$. However, this does not happen in the case of the considered cost function (5) because it includes a term penalizing the control effort.

8. Conclusions

The problem of amplitude-constrained self-tuning tracking LQG control is considered. An extension of the method given in [2] is presented to handle the control problem for constant or piecewise constant set-point. Stability and performance robustness of the LQG self-tuning control are shortly discussed. Second-order stable, unstable and non-minimum phase examples are analyzed and simulated. For stable open-loop systems, imposing hard constraints is allowable, however the constrained self-tuning responses are sluggish. In the case of unstable open-loop systems, it is generally not possible to achieve the closed-loop stability. However, this is possible in some special cases like noise-free (or noise-bounded) systems under some specified input constraint depending on the parametric uncertainty, initial conditions and magnitude of the set-point.

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